كلية الحاسبات والذكاء الإصطناعي

BS111

Probability and Statistics

Lecture 06

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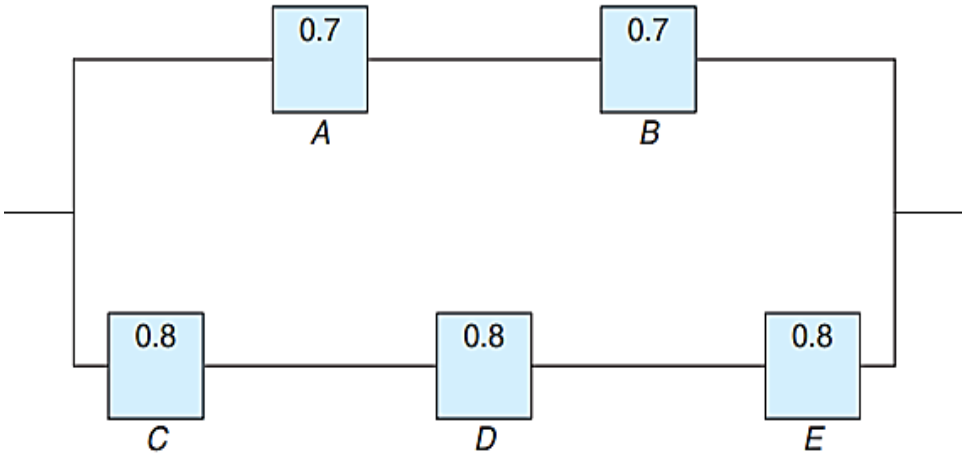
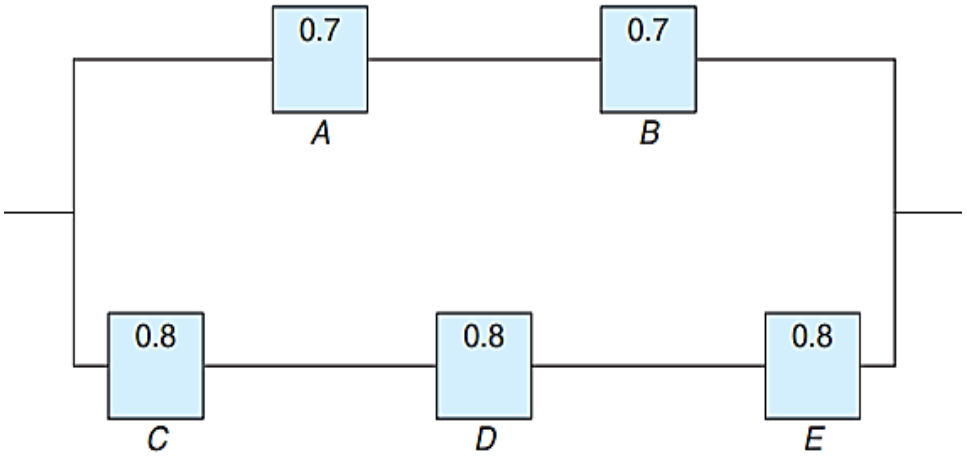
Quiz (1) – Q1

1. A circuit system is given in the following figure. Assume the components fail independently. What is the probability that the entire system works?



Quiz (1) – Q1

1. A circuit system is given in the following figure. Assume the components fail independently. What is the probability that the entire system works?



UP: (0.7) (0.7) = 0.49

Down: (0.8) (0.8) (0.8) = 0.512

$P(W) = 1 - [(0.51)\ (0.488)] = 0.75112$

Quiz (1) – Q2

2. If a multiple-choice test consists of 6 questions, each with 4 possible answers of which only 1 is correct. In how many ways can a student check off one answer to each question and get all the answers wrong?

Quiz (1) – Q2

2. If a multiple-choice test consists of 6 questions, each with 4 possible answers of which only 1 is correct. In how many ways can a student check off one answer to each question and get all the answers wrong?

Ways to get all the answers wrong $= (3)^6 = 729$

Quiz (1) – Q3

3. If A, and B are two independent events. And $P(A) = 0.2$, $P(A \cap B) = 0.06$, determine the following probabilities:

a) $P(A|B)$

b) $P(B|A)$

Quiz (1) – Q3

3. If A, and B are two independent events. And $P(A) = 0.2$, $P(A \cap B) = 0.06$, determine the following probabilities:

a) $P(A|B)$

$$P(A|B) = P(A) = 0.2$$

b) $P(B|A)$

$$P(B|A) = P(\mathrm{B}) = \frac{P(A \cap B)}{P(A)} = \frac{0.06}{0.2} = 0.3$$

Quiz (1) – Q4

4. In how many ways can 3 boys and 4 girls sit in a row?

Quiz (1) – Q4

4. In how many ways can 3 boys and 4 girls sit in a row?

$$\text{number of ways} = \frac{n!}{n_1!\, n_2!} = \frac{7!}{3!\, 4!} = 35$$

Quiz (1) – Q5

5. The likely location of a mobile device in the home is as follows:

Adult bedroom: 0.10, Child bedroom: 0.20,

Office: 0.40, Other rooms: 0.30

(a) What is the probability that a mobile device is in a bedroom?

(b) What is the probability that it is not in a bedroom?

Quiz (1) – Q5

5. The likely location of a mobile device in the home is as follows:

Adult bedroom: 0.10, Child bedroom: 0.20,

Office: 0.40, Other rooms: 0.30

(a) What is the probability that a mobile device is in a bedroom?

$= P(Adult\ bedroom \cup Child\ bedroom) = 0.10 + 0.20 = 0.30$

(b) What is the probability that it is not in a bedroom?

$= 1 - 0.30 = 0.70$

Quiz (1) – Q6

6. How many distinct permutations can be made from the letters of the word COMPUTER?

Quiz (1) – Q6

6. How many distinct permutations can be made from the letters of the word COMPUTER?

$n! = 8! = 40{,}320$

Quiz (1) – Q7

7. If a multiple-choice test consists of 6 questions, each with 4 possible answers of which only 1 is correct. In how many different ways can a student check off one answer to each question?

Quiz (1) – Q7

7. If a multiple-choice test consists of 6 questions, each with 4 possible answers of which only 1 is correct. In how many different ways can a student check off one answer to each question?

$= (4)^6 = 4{,}096$

Chapter 2: Random Variable

- Discrete Random Variables.
- Probability Mass Functions.
- Cumulative Distribution Functions.
- Discrete R.V. (Mean and Variance).
- Continuous Random Variables.
- Probability Density Functions.
- Continuous R.V. (Mean and Variance).
- Joint Probability Distributions.

Prob. Density Functions (1/2)

Rev. Example:

Suppose that the error in the reaction temperature, in °C (Celsius), for a controlled laboratory experiment is a continuous random variable X having the probability density function

$$f(x) = \begin{cases} \frac{x^2}{3}, & -1 < x < 2, \\ 0, & \text{elsewhere.} \end{cases}$$

(a) Verify that $f(x)$ is a density function.

(b) Find $P(0 < X \leq 1)$.

Prob. Density Functions (2/2)

Rev. Example – Answer (1/2)

Check the probability density function:

$$\int_{-1}^{2} \frac{x^2}{3}\, dx$$

$$\int_{-1}^{2} \frac{x^2}{3}\, dx = \frac{x^3}{9}\Big|_{-1}^{2} = \left(\frac{(2)^3}{9}\right) - \left(\frac{(-1)^3}{9}\right) = \frac{8}{9} + \frac{1}{9} = \mathbf{1}$$

Prob. Density Functions (2/2)

Rev. Example – Answer (2/2)

2) $\boldsymbol{P(0 < X \leq 1)}$

$$\boldsymbol{P(0 < X \leq 1)} = \int_0^1 \frac{x^2}{3}\, \boldsymbol{dx} = \frac{x^3}{9}\Big|_0^1$$

$$= \left(\frac{(1)^3}{9}\right) - \left(\frac{(0)^3}{9}\right) = \frac{1}{9} + \frac{0}{9} = \boldsymbol{\frac{1}{9}}$$

Cumulative Distribution Fun. (1/4)

Cumulative Distribution Function

The **cumulative distribution function** of a continuous random variable X is

$$F(x) = P(X \leq x) = \int_{-\infty}^{x} f(u)\,du$$

for $-\infty < x < \infty$.

Probability Density Function from the Cumulative Distribution Function

Given $F(x)$,

$$f(x) = \frac{dF(x)}{dx}$$

as long as the derivative exists.

Cumulative Distribution Fun. (2/4)

Cumulative Distribution Function

The **cumulative distribution function** of a continuous random variable X is

$$F(x) = P(X \leq x) = \int_{-\infty}^{x} f(u)\,du$$

for $-\infty < x < \infty$.

$$P(a < X < b) = F(b) - F(a) \text{ and } f(x) = \frac{dF(x)}{dx},$$

if the derivative exists.

Cumulative Distribution Fun. (3/4)

Example1:

Find the cumulative distribution function $F(x)$ and use it to evaluate $P(0 < X \leq 1)$.

$$f(x) = \begin{cases} \frac{x^2}{3}, & -1 < x < 2, \\ 0, & \text{elsewhere.} \end{cases}$$

Cumulative Distribution Fun. (4/4)

Example1 – Answer (1/3)

Find the cumulative distribution function $F(x)$

$$F(x) = \int_{-\infty}^{x} \frac{u^2}{3}\, du$$

$$\frac{x^2}{3}, \quad -1 < x < 2,$$

Cumulative Distribution Fun. (4/4)

Example1 – Answer (1/3)

Find the cumulative distribution function $F(x)$

$$F(x) = \int_{-\infty}^{x} \frac{u^2}{3}\, du = \int_{-1}^{x} \frac{u^2}{3}\, du$$

$$\frac{x^2}{3}, \quad -1 < x < 2,$$

Cumulative Distribution Fun. (4/4)

Example1 – Answer (2/3)

Find the cumulative distribution function $F(x)$

$$F(x) = \int_{-\infty}^{x} \frac{u^2}{3}\,\boldsymbol{du} = \int_{-1}^{x} \frac{u^2}{3}\,\boldsymbol{du} = \frac{u^3}{9}\Big|_{-1}^{x} = \left(\frac{(x)^3}{9}\right) - \left(\frac{(-1)^3}{9}\right)$$

$$= \frac{x^3}{9} + \frac{1}{9} = \frac{x^3+1}{9}$$

Cumulative Distribution Fun. (4/4)

Example1 – Answer (2/3)

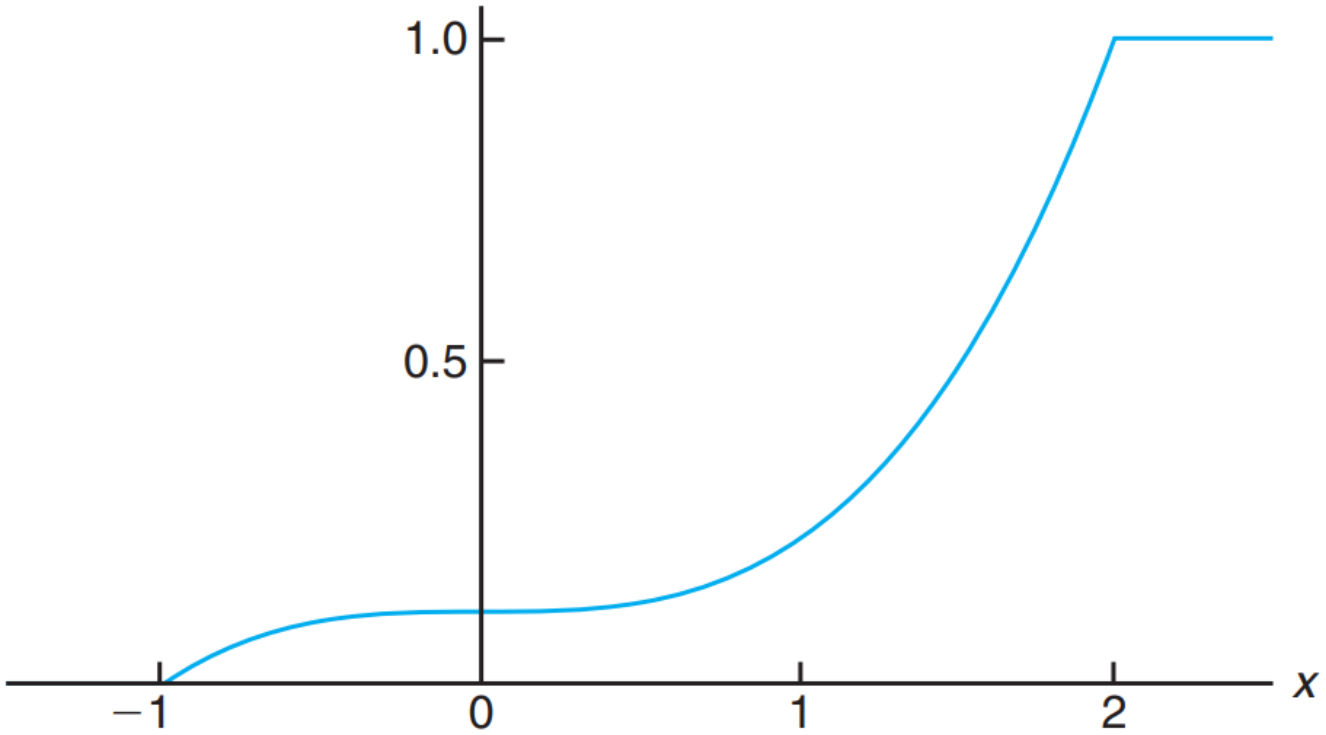
Find the cumulative distribution function $F(x)$

$$F(x) = \begin{cases} 0, & x < -1, \\ \frac{x^3+1}{9}, & -1 \leq x < 2, \\ 1, & x \geq 2. \end{cases}$$

Cumulative Distribution Fun. (4/4)

Example1 – Answer (2/3)

Find the cumulative distribution function $F(x)$



Cumulative Distribution Fun. (4/4)

Example1 – Answer (3/3)

Find the cumulative distribution function $F(x)$ and use it to evaluate $P(0 < X \leq 1)$.

$$F(x) = \frac{x^3 + 1}{9}$$

$$P(0 < X \leq 1) = F(1) - F(0) = \frac{2}{9} - \frac{1}{9} = \frac{1}{9}$$

Mean and Variance (1/3)

Mean and Variance

Suppose that X is a continuous random variable with probability density function $f(x)$.

mean or **expected value** of X, denoted as μ or $E(X)$, is

$$\mu = E(X) = \int_{-\infty}^{\infty} xf(x)dx$$

The **variance** of X, denoted as $V(X)$ or σ^2, is

$$\sigma^2 = V(X) = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)dx = \int_{-\infty}^{\infty} x^2 f(x)dx - \mu^2$$

The **standard deviation** of X is $\sigma = \sqrt{\sigma^2}$.

Mean and Variance (2/3)

Example1:

Find the expected value of X, $E(X)$ and the variance $V(X)$.

$$f(x) = \begin{cases} \frac{x^2}{3}, & -1 < x < 2, \\ 0, & \text{elsewhere.} \end{cases}$$

Mean and Variance (3/3)

Example1 – Answer (1/3)

Find the expected value of X, $E(X)$ and the variance $V(X)$.

$$f(x) = \begin{cases} \frac{x^2}{3}, & -1 < x < 2, \\ 0, & \text{elsewhere.} \end{cases}$$

$$E(X) = \int_{-\infty}^{\infty} x f(x)\, dx$$

Mean and Variance (3/3)

Example1 – Answer (1/3)

Find the expected value of X, $E(X)$ and the variance $V(X)$.

$$f(x) = \begin{cases} \frac{x^2}{3}, & -1 < x < 2, \\ 0, & \text{elsewhere.} \end{cases}$$

$$E(X) = \int_{-\infty}^{\infty} x f(x)\, dx = \int_{-1}^{2} \frac{x^3}{3}\, dx = \left. \frac{x^4}{12} \right|_{-1}^{2}$$

Mean and Variance (3/3)

Example1 – Answer (1/3)

Find the expected value of X, $E(X)$ and the variance $V(X)$.

$$f(x) = \begin{cases} \frac{x^2}{3}, & -1 < x < 2, \\ 0, & \text{elsewhere.} \end{cases}$$

$$E(X) = \int_{-1}^{2} \frac{x^3}{3}\,dx = \left.\frac{x^4}{12}\right|_{-1}^{2} = \frac{(2)^4}{12} - \frac{(-1)^4}{12} = \frac{15}{12}$$

Mean and Variance (3/3)

Example1 – Answer (2/3)

Find the expected value of X, $E(X)$ and the variance $V(X)$.

$$f(x) = \begin{cases} \frac{x^2}{3}, & -1 < x < 2, \\ 0, & \text{elsewhere.} \end{cases}$$

$$E(X^2) = \int_{-1}^{2} \frac{x^4}{3}\,dx = \frac{x^5}{15}\Big|_{-1}^{2} = \frac{(2)^5}{15} - \frac{(-1)^5}{15} = \frac{33}{15}$$

Mean and Variance (3/3)

Example1 – Answer (3/3)

Find the expected value of X, $E(X)$ and the variance $V(X)$.

$$E(X) = \frac{15}{12}$$

$$E(X^2) = \frac{33}{15}$$

$$V(X) = E(X^2) - \big(E(X)\big)^2 = \frac{33}{15} - \left(\frac{15}{12}\right)^2 = 0.6375$$

Mean and Variance (3/3)

Example1 – Answer (3/3)

Find the expected value of X, $E(X)$ and the variance $V(X)$.

$$E(X) = \frac{15}{12}$$

$$E(X^2) = \frac{33}{15}$$

Standard Deviation (σ)
$= \sqrt{0.6375} = 0.798$

$$V(X) = E(X^2) - \big(E(X)\big)^2 = \frac{33}{15} - \left(\frac{15}{12}\right)^2 = 0.6375$$

Joint Prob. Distributions (1/9)

Definition:

If X and Y are two random variables, the probability distribution that defines their simultaneous behavior is called a joint probability distribution.

Joint Prob. Distributions (2/9)

If X and Y are discrete random variables:

Joint Probability Mass Function

The **joint probability mass function** of the discrete random variables X and Y, denoted as $f_{XY}(x, y)$, satisfies

(1) $f_{XY}(x, y) \geq 0$

(2) $\sum_{X}\sum_{Y} f_{XY}(x, y) = 1$

(3) $f_{XY}(x, y) = P(X = x, Y = y)$

Joint Prob. Distributions (3/9)

Example1:

A coin is tossed two times. Calculate the joint probability mass function of the discrete random variables X and Y, where X denote the number of heads appear and Y denote the number of tails appear.

Joint Prob. Distributions (4/9)

Example1 – Answer (1/2)

$S = \{HH, HT, TH, TT\}$

X	2	1	1	0
Y	0	1	1	2

Joint Prob. Distributions (4/9)

Example1 – Answer (2/2)

$S = \{HH, HT, TH, TT\}$

X	2	1	1	0
Y	0	1	1	2

y \ x	**0**	**1**	**2**
0	0	0	$1/4$
1	0	$2/4$	0
2	$1/4$	0	0

Joint Prob. Distributions (5/9)

Example2:

A coin is tossed two times. Calculate the joint probability mass function of the discrete random variables X and Y, where X denote the number of heads appear and Y denote the number of tails appear.

Find:

1) $f_{XY}(1,2) = P(X = 1,\ Y = 2)$
2) $f_{XY}(2,0) = P(X = 2,\ Y = 0)$
3) $P(X = 1,\ Y \leq 2)$
4) $P(Y = 2)$

Joint Prob. Distributions (6/9)

Example2 – Answer (1/4)

y \ x	**0**	**1**	**2**
0	0	0	1/4
1	0	2/4	0
2	1/4	0	0

Find:

1) $f_{XY}(1,2) = P(X = 1,\ Y = 2)$

Joint Prob. Distributions (6/9)

Example2 – Answer (1/4)

y \ x	**0**	**1**	**2**
0	0	0	1/4
1	0	2/4	0
2	1/4	0	0

Find:

1) $f_{XY}(1,2) = P(X = 1,\ Y = 2) = 0$

Joint Prob. Distributions (6/9)

Example2 – Answer (2/4)

y \ x	**0**	**1**	**2**
0	0	0	1/4
1	0	2/4	0
2	1/4	0	0

Find:

2) $f_{XY}(2,0) = P(X = 2,\ Y = 0)$

Joint Prob. Distributions (6/9)

Example2 – Answer (2/4)

y \ x	**0**	**1**	**2**
0	0	0	1/4
1	0	2/4	0
2	1/4	0	0

Find:

2) $f_{XY}(2,0) = P(X = 2,\ Y = 0) = 1/4$

Joint Prob. Distributions (6/9)

Example2 – Answer (3/4)

y \ x	**0**	**1**	**2**
0	0	0	1/4
1	0	2/4	0
2	1/4	0	0

Find:

3) $P(X = 1,\ Y \leq 2)$

Joint Prob. Distributions (6/9)

Example2 – Answer (3/4)

y \ x	**0**	**1**	**2**
0	0	0	1/4
1	0	2/4	0
2	1/4	0	0

Find:

3) $P(X = 1,\ Y \leq 2) = 0 + \frac{2}{4} + 0 = \frac{2}{4}$

Joint Prob. Distributions (6/9)

Example2 – Answer (4/4)

y \ x	**0**	**1**	**2**
0	0	0	1/4
1	0	2/4	0
2	1/4	0	0

Find:

4) $P(Y = 2)$

Joint Prob. Distributions (6/9)

Example2 – Answer (4/4)

y \ x	**0**	**1**	**2**
0	0	0	1/4
1	0	2/4	0
2	1/4	0	0

Find:

4) $P(Y = 2) = \frac{1}{4} + 0 + 0 = \frac{1}{4}$

Joint Prob. Distributions (7/9)

Marginal Probability Distributions

The marginal distributions of the random variable X alone is:

$$f_X(x) = \sum_y f_{XY}(x, y)$$

The marginal distributions of the random variable Y alone is:

$$f_Y(y) = \sum_x f_{XY}(x, y)$$

Joint Prob. Distributions (8/9)

Example3:

A coin is tossed two times. Calculate the joint probability mass function of the discrete random variables X and Y, where X denote the number of heads appear and Y denote the number of tails appear.

Find:

1) $f_X(x)$

2) $f_Y(y)$

Joint Prob. Distributions (9/9)

Example3 – Answer (1/4)

y \ x	**0**	**1**	**2**
0	0	0	1/4
1	0	2/4	0
2	1/4	0	0

Find:

1) $f_X(x)$

Joint Prob. Distributions (9/9)

Example3 – Answer (1/4)

y \ x	**0**	**1**	**2**
0	0	0	1/4
1	0	2/4	0
2	1/4	0	0
$f_X(x)$	**1/4**	**2/4**	**1/4**

Find:

1) $f_X(x)$

Joint Prob. Distributions (9/9)

Example3 – Answer (2/4)

x	0	1	2
$f_X(x)$	1/4	2/4	1/4

Find:

1) $f_X(x)$

Joint Prob. Distributions (9/9)

Example3 – Answer (3/4)

y \ x	**0**	**1**	**2**
0	0	0	1/4
1	0	2/4	0
2	1/4	0	0

Find:

2) $f_Y(y)$

Joint Prob. Distributions (9/9)

Example3 – Answer (3/4)

y \ x	**0**	**1**	**2**	$f_Y(y)$
0	0	0	1/4	**1/4**
1	0	2/4	0	**2/4**
2	1/4	0	0	**1/4**

Find:

2) $f_Y(y)$

Joint Prob. Distributions (9/9)

Example3 – Answer (4/4)

y	0	1	2
$f_Y(y)$	1/4	2/4	1/4

Find:

2) $f_Y(y)$

Video Lectures

All Lectures: https://www.youtube.com/playlist?list=PLxIvc-MGOs6gW9SgkmoxE5w9vQkID1_r-

Lecture #6: https://www.youtube.com/watch?v=crwvc-eW8t8&list=PLxIvc-MGOs6gW9SgkmoxE5w9vQkID1_r-&index=7

Thank You

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